

Deconfinement transition in $SU(N)$ theories from perturbation theory

U. Reinosa^a, J. Serreau^b, M. Tissier^c, N. Wschebor^d

^a*Centre de Physique Théorique, Ecole Polytechnique, CNRS, 91128 Palaiseau Cedex, France.*

^b*Astro-Particule et Cosmologie (APC), CNRS UMR 7164, Université Paris 7 - Denis Diderot
10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France.*

^c*LPTMC, Laboratoire de Physique Théorique de la Matière Condensée, CNRS UMR 7600,
Université Pierre et Marie Curie, boîte 121, 4 pl. Jussieu, 75252 Paris Cedex 05, France.*

^d*Instituto de Física, Facultad de Ingeniería, Universidad de la República,
J.H.y Reissig 565, 11000 Montevideo, Uruguay.*

Abstract

We consider a simple massive extension of the Landau-DeWitt gauge for $SU(N)$ Yang-Mills theory. We compute the corresponding one-loop effective potential for a temporal background gluon field at finite temperature. At this order the background field is simply related to the Polyakov loop, the order parameter of the deconfinement transition. Our perturbative calculation correctly describes a quark confining phase at low temperature and a phase transition of second order for $N = 2$ and weakly first order for $N = 3$. Our estimates for the transition temperatures are in qualitative agreement with values from lattice simulations or from other continuum approaches. Finally, we discuss the effective gluon mass parameter in relation to the Gribov ambiguities of the Landau-DeWitt gauge.

Keywords: Yang-Mills theory, finite temperature QFT, deconfinement transition, Gribov ambiguities

1. Introduction

At finite temperature, $SU(N)$ Yang-Mills theories present a phase transition associated with the spontaneous breaking of the Z_N center symmetry [1] with a transition temperature roughly of the order of the intrinsic scale of the theory [2]. The Polyakov loop provides a relevant order parameter for this transition which can be related to the free energy of an isolated static quark [3]. At high temperature, where center symmetry is broken, the Polyakov-loop admits a non zero value corresponding to a finite free energy and thus to a quark deconfined phase. As the temperature is lowered, center symmetry is eventually restored, the Polyakov loop vanishes and the free energy becomes infinite signaling the presence of a quark confining phase. It is widely accepted that this phenomenon, which has its origins in the infrared (IR) dynamics of the theory, is intrinsically non-perturbative. This is based on the failure of standard perturbative techniques for energy scales of the order of the transition temperature. Existing calculations involve either direct lattice simulations [1, 4, 5, 6, 7] or nonperturbative semi-analytical techniques such as the functional renormalization group (FRG) [8, 9, 10, 11], Schwinger-Dyson equations [12, 13, 14, 15], the Hamiltonian approach of [16], or a two-particle-irreducible in-

spired approach [17, 11]. Another interesting approach is that based on the (refined) Gribov-Zwanziger action [18, 19]. To our knowledge, it has been used to study the behavior of thermodynamic quantities across the transition [20, 21, 22, 23] but, so far, not to investigate the issue of center-symmetry restoration at low temperatures.

In the present Letter, we show that the main aspects of the Z_N transition can be described by means of a slightly modified perturbative expansion. Our approach is based on a series of recent developments concerning the description of the infrared sector of gauge-fixed Yang-Mills theories [24, 25, 26, 27, 28, 29, 30]. Lattice calculations of gluon and ghost-antighost correlators in the Landau gauge, both in the vacuum and at finite temperature, show that at low momenta, the gluon behaves as a massive field whereas the ghost and antighost remain massless. This has motivated two of us to study a simple massive extension of the Faddeev-Popov Landau gauge Lagrangian [24, 25], which is a special case of the Curci-Ferrari model [31] also known as the Fradkin-Tyutin model [32]. A one-loop calculation reproduces the lattice data for the vacuum two-point correlators with very good accuracy all the way from high momenta down to the deep infrared regime. The effective gluon mass also provides an infrared regulator which allows

for infrared safe renormalization group trajectories with no Landau pole. A similar one-loop calculation of the vacuum three-point correlators in the Landau gauge has been performed recently [27] and gives equally good results, although the lattice data are more noisy. This demonstrates that the dominant infrared physics (in the vacuum and in the Landau gauge) is efficiently captured by a simple effective gluon mass.

A qualitative but instructive analogy can be made with the physics of the Kosterlitz-Thouless transition in the XY model in statistical physics [33, 34]. In terms of usual spin waves, the physics of the transition is intrinsically nonperturbative: perturbation theory shows no sign of the transition at all orders. Alternatively, taking explicitly into account the vortex excitations of the XY model leads to a valid description of the transition by perturbative means. The validity of a perturbative description crucially depends on having a physically sound reference action.

The analogy with (gauge-fixed) Yang-Mills theories lies in the fact that the Faddeev-Popov action completely ignores the existence of Gribov ambiguities and is, at best, a valid description at asymptotically high energies, where the latter play a negligible role. In this formulation, the description of low energy phenomena is a problem of notorious nonperturbative nature. Instead, the massive extension of the Faddeev-Popov Lagrangian seems to provide a more efficient starting point for the (perturbative) description of low energy phenomena. This may be due to the fact that a complete gauge fixing procedure, which consistently deals with the Gribov issue, is likely to explicitly break the BRST symmetry and thus to induce (effective) BRST breaking terms. The simplest such term consistent with the requirement of renormalizability and locality is a gluon mass term.

Such qualitative ideas can be given more solid theoretical foundations. Explicit examples in the Landau gauge are the (refined) Gribov-Zwanziger approach [18, 19], where one restricts the path integral to the first Gribov region, or the quantization procedure of [26], where one performs a particular average over Gribov copies. Both approaches induce a soft breaking of the BRST symmetry.¹ In the latter case, the resulting gauge-fixed Yang-Mills action has been shown to be perturbatively equivalent to the Curci-Ferrari-Fradkin-Tyutin (CFFT) model for what concerns the calculation of gluon and ghost-antighost correlators. In this context, the (bare) gluon mass appears as a gauge-fixing pa-

rameter which lifts the degeneracy between the Gribov copies. The one-loop calculations of Yang-Mills correlators mentioned above can be viewed as genuine applications of perturbation theory in this class of gauges.

More recently, we have investigated to what extent the above results hold at finite temperature [29] by comparing to various lattice results [35, 36, 37, 38]. We have performed a one-loop calculation of the gluon (electric and magnetic) and ghost-antighost propagators, fitting the mass and coupling parameters against the lattice data for each temperature. We found a quantitative agreement for the ghost and magnetic gluon propagators, demonstrating that a simple mass term is enough to capture the corresponding physics all the way to deep infrared momenta. As for the electric gluon propagator, the results are quantitatively good except near the transition temperature, where the agreement is only qualitative. It must be mentioned though that lattice data are pretty noisy in this region [35, 39, 40]. These results are similar to those from other continuum approaches [41].

However, an important physical ingredient is missing in these continuum calculations, namely the explicit Z_N symmetry and the existence of an associated order parameter. In fact, this symmetry is explicitly broken in the Landau gauge and, although this should not affect physical observables in an exact calculation, it questions the ability of approximate approaches to capture the physics of the transition. Besides, this may also explain the poor convergence of lattice results for, say, the Debye mass near the transition temperature [40].

It is, therefore, desirable to devise an approach where the Z_N symmetry can be explicitly accounted for both on the lattice and in (approximate) continuum calculations. Background field gauge (BFG) methods provide an efficient way to go since the center symmetry is explicit at the quantum level and easy to maintain in approximation schemes. In Refs. [9, 10, 11], the authors have studied the Z_N deconfinement phase transition by means of FRG techniques in the BFG formalism in the Landau-DeWitt (LDW) gauge, the background field generalization of the Landau gauge. This approach correctly predicts the order of the phase transition for $N = 2$ and $N = 3$ and gives reasonable transition temperatures.

In the present Letter, we follow this path and investigate whether the perturbative approach described above, based on a simple gluon mass, is able to capture the physics of the deconfinement transition in the BFG formalism. More precisely, we implement a simple massive extension of the LDW gauge. This is in fact equivalent to the CFFT model in the presence of a source associated with a composite operator $s\bar{s}A_\mu$, where s and \bar{s} denote appropriate BRST and antiBRST variations

¹The minimal Landau gauge on the lattice, where one selects a particular Gribov copy, provides another example, although less explicit since it has no known continuum formulation.

[42, 43]. The quantum action in the presence of this source satisfies a new Ward identity [42] which actually corresponds to the background field gauge symmetry in the BFG formulation [43]. As we shall describe below, this symmetry is what allows one to keep track of the underlying center symmetry. In the BFG formalism, observables such as the thermodynamic pressure or the Polyakov loop can be computed unambiguously by evaluating their functional expression at the minimum of a particular background field potential.

We compute this potential at one-loop order in the massive version of the LDW gauge and follow its minimum as a function of the temperature. We probe the Z_N symmetry by computing the Polyakov loop evaluated at this minimum. At the order of approximation considered here, it coincides with the mean field expression considered in previous FRG approaches. Although, this identification does not hold beyond leading order, the mean field Polyakov loop is expected to provide an equally valid order parameter [8, 9].²

Thanks to the gluon mass, the gluon loop contribution is suppressed at low temperatures and the potential is dominated by the massless ghost loop. This leads to a center symmetric minimum, both in SU(2) and SU(3), hence to a confined phase, in accordance with the results of [9]. At higher temperatures, a non-trivial, center symmetry breaking minimum develops. We find a second order transition in SU(2) and a first order one in SU(3), as expected, with transition temperatures controlled by the gluon mass. We estimate the latter from fits of lattice data for the vacuum gluon propagator in the Landau gauge. Our one-loop calculation essentially reproduces the results of the FRG approach of Ref. [9]. We emphasize that the present calculation can be systematically improved by computing higher order contributions.

Finally, we discuss the relation of the gluon mass with the issue of Gribov ambiguities in the LDW gauge. The quantization procedure proposed in [26] in the Landau gauge can be generalized to the LDW gauge in a straightforward way. The resulting gauge-fixed Yang-Mills action is perturbatively equivalent to the massive model discussed in the core of the paper as far as gluon and ghost-antighost correlators are concerned. As in the Landau gauge, the gluon mass term appears as a tool to control the Gribov problem and completely fixes the gauge.

²This has been proved for the SU(2) theory in the Polyakov gauge in [8]. However, to our knowledge, no proof exists in the LDW gauge. Alternative order parameters have also been proposed; see, e.g., [44].

2. General set-up

2.1. Massive background field action and Z_N symmetry

We consider a SU(N) Yang-Mills theory in d dimensions with Euclidean time $\tau \in [0, \beta]$, where $\beta = 1/T$ is the inverse temperature. The classical action reads

$$S_{\text{YM}}[A] = \frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a, \quad (1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$, with g the coupling constant, and $\int_x = \int_0^\beta d\tau \int d^{d-1}x$. Our convention for the SU(N) generators is $\text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$ and $[t^a, t^b] = i f^{abc} t^c$. We define matrix fields as $\varphi = \varphi^a t^a$.

We quantize the theory using the background field method [45], which introduces an *a priori* arbitrary background field configuration \bar{A}_μ through a modified gauge-fixing condition. In terms of $a_\mu = A_\mu - \bar{A}_\mu$, the LDW gauge condition reads

$$\bar{D}_\mu a_\mu^a = 0, \quad (2)$$

where $\bar{D}_\mu^{ab} = \partial_\mu \delta^{ab} + g f^{acb} \bar{A}_\mu^c$. Here and in the following, we use the notation $\bar{D}_\mu \varphi^a = \bar{D}_\mu^{ab} \varphi^b$. The standard Faddeev-Popov gauge-fixed action reads [43]

$$S_{\bar{A}}^{\text{gf}} = \int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{D}_\mu \bar{c}^a D_\mu c^a + i h^a \bar{D}_\mu a_\mu^a \right\}, \quad (3)$$

with a (real) Nakanishi-Lautrup field h and ghosts and antighost fields c and \bar{c} . Note that, in terms of the field a_μ , one has $F_{\mu\nu}^a = \bar{F}_{\mu\nu}^a + \bar{D}_\mu a_\nu^a - \bar{D}_\nu a_\mu^a + g f^{abc} a_\mu^b a_\nu^c$, with $\bar{F}_{\mu\nu}^a$ the background field strength tensor, and $\bar{D}_\mu^{ab} = \bar{D}_\mu^{ab} + g f^{acb} a_\mu^c$.

The action (3) has the obvious symmetry

$$S_{\bar{A}}^{\text{gf}}[\varphi] = S_{\bar{A}U}^{\text{gf}}[U\varphi U^{-1}] \quad (4)$$

where $\varphi = (a, c, \bar{c}, h)$, U is a local SU(N) matrix and

$$\bar{A}_\mu^U = U \bar{A}_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1}. \quad (5)$$

Here, we consider a simple massive extension of (3) which preserves the symmetry:

$$S_{\bar{A}} = \int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{m^2}{2} a_\mu^a a_\mu^a + \bar{D}_\mu \bar{c}^a D_\mu c^a + i h^a \bar{D}_\mu a_\mu^a \right\}. \quad (6)$$

It is easy to check that the linear symmetry (4) implies the corresponding one at the level of the (quantum) effective action Γ [45]:

$$\Gamma_{\bar{A}}[\varphi] = \Gamma_{\bar{A}U}[\varphi U^{-1}], \quad (7)$$

where the collection of fields in φ are now to be understood as average values in the presence of sources.³ The symmetry (4) is trivially preserved in perturbation theory. In particular, it encodes the relevant center symmetry governing the deconfinement transition at finite temperature.

2.2. Observables and the effective potential

Before embarking in actual calculations, we recall how gauge-invariant observables, such as the thermodynamic pressure or the Polyakov loop to be discussed below, can be conveniently obtained by choosing a particular background field $\bar{A} = \bar{A}_{\min}$ which minimizes a specific background field potential defined below. For the sake of the argument, let us consider the case of vanishing sources for the fields c , \bar{c} and h . We thus consider the effective action $\Gamma_{\bar{A}}[a]$ for the field a and a given background \bar{A} .

A physical (gauge-invariant) observable O can be obtained as the value of a given functional $O_{\bar{A}}[a]$ evaluated for a at the absolute minimum of the functional $\Gamma_{\bar{A}}[a]$:

$$O = O_{\bar{A}}[a_{\bar{A}}^{\min}], \quad (8)$$

where $a_{\bar{A}}^{\min}$ satisfies

$$\Gamma_{\bar{A}}[a_{\bar{A}}^{\min}] \leq \Gamma_{\bar{A}}[a] \quad \forall a. \quad (9)$$

Note that, by definition, physical observables are independent of the gauge-fixing condition and thus of \bar{A} . In particular, the effective action evaluated at its absolute minimum is directly related to the partition function Z and is thus gauge invariant:

$$-\ln Z = \Gamma_{\bar{A}}[a_{\bar{A}}^{\min}] \quad \forall \bar{A}. \quad (10)$$

Taking advantage of the background field independence of observables, it is judicious to choose $\bar{A} = \bar{A}_{\min}$, defined as $a_{\bar{A}_{\min}}^{\min} = 0$. We then have

$$O = O_{\bar{A}_{\min}}[0]. \quad (11)$$

Indeed, consider the following functional of the background field⁴

$$\tilde{\Gamma}[\bar{A}] = \Gamma_{\bar{A}}[0], \quad (12)$$

which is invariant under the gauge transformation (5): $\tilde{\Gamma}[\bar{A}] = \tilde{\Gamma}[\bar{A}^U]$, as follows from Eq. (7). One easily

proves that \bar{A}_{\min} , defined above, is an absolute minimum of $\tilde{\Gamma}[\bar{A}]$:

$$\tilde{\Gamma}[\bar{A}_{\min}] = \Gamma_{\bar{A}_{\min}}[a_{\bar{A}_{\min}}^{\min}] = \Gamma_{\bar{A}}[a_{\bar{A}}^{\min}] \leq \Gamma_{\bar{A}}[0] = \tilde{\Gamma}[\bar{A}] \quad \forall \bar{A}. \quad (13)$$

Here, the first equality just states the definitions of $\tilde{\Gamma}$ and of \bar{A}_{\min} , the second one follows from the gauge invariance of the partition function, Eq. (10), and the inequality is just Eq. (9) taken for $a = 0$.

For our present purposes, it is sufficient to consider homogeneous background fields in the temporal direction. We thus consider the background field potential

$$V^{\text{SU}(N)}(T, \beta g \bar{A}) = \frac{\tilde{\Gamma}[\bar{A}]}{\beta \Omega}, \quad (14)$$

where Ω is the spatial volume. The thermodynamic pressure is then simply given by

$$p = -V^{\text{SU}(N)}(T, \beta g \bar{A}_{\min}), \quad (15)$$

whereas the Polyakov loop is obtained as

$$\ell = \frac{1}{N} \text{tr} \left\{ P \exp \left\{ ig \int_0^\beta d\tau [\bar{A}_0^{\min} + a_0(\tau, \mathbf{x})] \right\} \right\}_{\min} \quad (16)$$

where P denotes the path ordering (operators are ordered from left to right according to the decreasing value of their time argument) and where the subscript 'min' means that the average must be evaluated for $\langle a \rangle = a_{\bar{A}_{\min}}^{\min} = 0$.

3. The background field potential at one loop

We consider a constant temporal background field $\bar{A}_\mu(x) = \bar{A}_0 \delta_{\mu 0}$. The Hermitian matrix \bar{A}_0 can be diagonalized by means of a global $\text{SU}(N)$ rotation and we can thus, with no loss of generality, consider the case where \bar{A}_0 belongs to the Cartan subalgebra of the gauge group: $A_0 \rightarrow A_0^a t_C^a$ with $[t_C^a, t_C^b] = 0$.

At lowest non-trivial (one-loop) order in a perturbative expansion, the potential (14) reads

$$V^{\text{SU}(N)}(T, r_a) = \frac{1}{\beta \Omega} \left\{ \frac{1}{2} \text{Tr} \text{Ln} \Delta_{a,h}^{-1} - \text{Tr} \text{Ln} \Delta_{c,\bar{c}}^{-1} \right\}, \quad (17)$$

where $r_a = \beta g \bar{A}_0^a$ and where $\Delta_{a,h}^{-1}$ and $\Delta_{c,\bar{c}}^{-1}$ denote the tree-level inverse propagator in the (a, h) and (c, \bar{c}) sectors respectively. At the same order of approximation, the Polyakov loop (16) assumes its tree-level expression

$$\ell = \frac{1}{N} \text{tr} [\exp(ir_a t^a)]. \quad (18)$$

where it is understood that the right-hand side must be evaluated at the absolute minimum of the potential (17). In the following, we compute explicitly the potential and the Polyakov loop in the cases $N = 2$ and $N = 3$.

³In particular h is now purely imaginary.

⁴Notice that $\tilde{\Gamma}[\bar{A}]$ is *not* a Legendre transform.

3.1. Second order transition for $N = 2$

The Cartan algebra has one single direction, say $a = 3$ and we note $r_3 = r$. As discussed in the Appendix, the background field potential is 2π -periodic in r as a result of the Z_2 center symmetry implies. Together with charge-conjugation invariance, this implies

$$V^{\text{SU}(2)}(T, \pi + r) = V^{\text{SU}(2)}(T, \pi - r). \quad (19)$$

It is thus sufficient to study the potential in the interval $r \in [0, \pi]$ and we see that both $r = 0$ and $r = \pi$ are always extrema. The tree-level Polyakov loop (18) reads

$$\ell = \cos(r/2). \quad (20)$$

One concludes that $r \in [0, \pi[$ and $r = \pi$ correspond respectively to Z_2 -breaking and Z_2 -symmetric solutions.

The calculation of the one-loop expression (17) is straightforward. Introducing the notations $n^a = r^a/r$ and $R_\mu = rT\delta_{\mu 0}$, the covariant derivative \bar{D} reads, in momentum space,⁵

$$i\bar{D}_\mu^{ab} \rightarrow Q_\mu \mathcal{P}_0^{ab} + (Q + R)_\mu \mathcal{P}_+^{ab} + (Q - R)_\mu \mathcal{P}_-^{ab}, \quad (21)$$

where $Q_\mu = (\omega_n, \mathbf{q})$, with $\omega_n = 2\pi nT$ the Matsubara frequencies and where we introduced the set of orthogonal Hermitian projectors

$$\mathcal{P}_0^{ab} = n^a n^b, \quad \mathcal{P}_\pm^{ab} = \frac{1}{2}(\delta^{ab} - n^a n^b \pm i\epsilon^{acb} n^c), \quad (22)$$

which satisfy $\mathcal{P}_\eta \mathcal{P}_{\eta'} = \delta_{\eta\eta'} \mathcal{P}_\eta$ and $\sum_{\eta=0,\pm} \mathcal{P}_\eta = \mathbb{1}$.

One can consider each orthogonal color sector independently. The covariant derivative (21) is block-diagonal with one 1×1 block along the Cartan direction (\mathcal{P}_0) and one 2×2 block in the transverse directions (\mathcal{P}_\pm). Since in the action (6) the background field only appears in the covariant derivative (21), we see that in each color sector the calculation of the inverse propagators $\Delta_{a,h}^{-1}$ and $\Delta_{c,\bar{c}}^{-1}$ and thus of the one-loop expression (17) is formally equivalent to considering the case of an imaginary chemical potential $\mu = irT$ with corresponding $U(1)$ charges $\eta = 0, \pm$. Introducing the notation

$$T^d F_{\tilde{m}}(r) \triangleq T \sum_{n \in \mathbb{Z}} \int_{\mathbf{q}} \ln[(Q + R)^2 + m^2], \quad (23)$$

where $\tilde{m} = m/T$, $\int_{\mathbf{q}} = \int d^{d-1}q/(2\pi)^{d-1}$, and \triangleq means that we subtract a (divergent) term independent of T and

⁵Our sign convention for Fourier transforms is $f(x) = \int dk e^{-ikx} \tilde{f}(k)$ and similarly for discrete Fourier series in the compact Euclidean time direction.

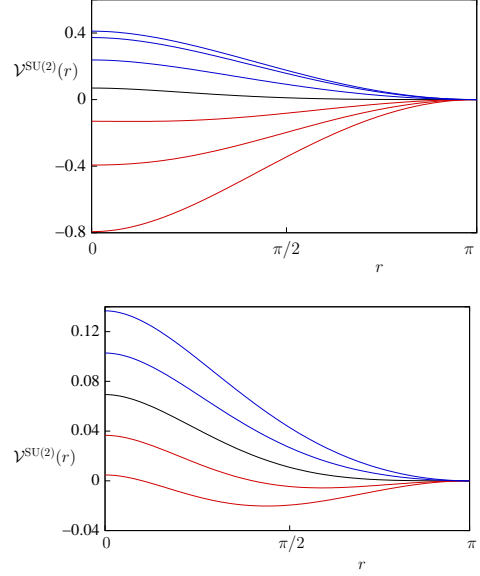


Figure 1: The background field potential (28) in $d = 4$, normalized to its value at $r = \pi$, for decreasing values of \tilde{m} (increasing temperatures) from top to bottom. The bottom figure is a close-up view around T_c . (color online: $T = T_c$ (black), $T < T_c$ (blue), $T > T_c$ (red))

r , we have

$$\frac{\text{Tr Ln } \Delta_{a,h}^{-1}}{\beta\Omega} \triangleq T^d \sum_{\eta=0,\pm} [(d-1)F_{\tilde{m}}(\eta r) + F_0(\eta r)], \quad (24)$$

$$\frac{\text{Tr Ln } \Delta_{c,\bar{c}}^{-1}}{\beta\Omega} \triangleq T^d \sum_{\eta=0,\pm} F_0(\eta r). \quad (25)$$

The Matsubara sum in (23) is easily evaluated as

$$F_{\tilde{m}}(r) = \int_{\mathbf{q}} \ln(1 + e^{-2\tilde{\epsilon}_q} - 2e^{-\tilde{\epsilon}_q} \cos r), \quad (26)$$

where $\tilde{\epsilon}_q = \sqrt{\mathbf{q}^2 + \tilde{m}^2}$. Introducing the function

$$\mathcal{W}_{\tilde{m}}(r) = \frac{1}{2}[(d-1)F_{\tilde{m}}(r) - F_0(r)], \quad (27)$$

our final expression for the dimensionless potential $\mathcal{V}^{\text{SU}(2)}(T, r) \equiv V^{\text{SU}(2)}(T, r)/T^d$ is

$$\mathcal{V}^{\text{SU}(2)}(T, r) \triangleq 2\mathcal{W}_{\tilde{m}}(r) + \mathcal{W}_{\tilde{m}}(0). \quad (28)$$

The two terms on the right-hand side come from the transverse and longitudinal color modes respectively. The potential (28) is plotted in Fig. 1.

In the high temperature limit $\tilde{m} \ll 1$, one has $\mathcal{W}_{\tilde{m}}(r) \approx (d-2)F_0(r)/2$ and we recover the standard Weiss potential [46]

$$\mathcal{V}^{\text{SU}(2)}(T, r) \approx \frac{d-2}{2} [2F_0(r) + F_0(0)], \quad (29)$$

with

$$F_0(r) = -2 \frac{\Omega_{d-1}}{(2\pi)^{d-1}} \Gamma(d-1) \sum_{n \geq 1} \frac{\cos nr}{n^d}, \quad (30)$$

where $\Omega_D = 2\pi^{D/2}/\Gamma(D/2)$. In particular, in $d = 4$, this reads

$$F_0^{d=4}(r) = \frac{(r-\pi)^4}{24\pi^2} - \frac{(r-\pi)^2}{12} + \frac{7\pi^2}{360} \quad \text{for } r \in [0, 2\pi]. \quad (31)$$

The function (29) presents a Z_2 -breaking minimum at $r = 0$. The extremum at $r = \pi$ is a maximum.

At low temperatures, the contribution from massive gluons in Eq. (27) is exponentially suppressed, leaving only the ghost contribution: $\mathcal{W}_{\tilde{m}}(r) \approx -F_0(r)/2$ and we get the inverted Weiss potential⁶

$$\mathcal{V}^{\text{SU}(2)}(T, r) \approx -\frac{1}{2} [2F_0(r) + F_0(0)]. \quad (32)$$

This exhibits a Z_2 -symmetric minimum at $r = \pi$. As the temperature increases, the potential flattens around the minimum $r = \pi$ and a new minimum eventually develops, as shown in Fig. 1. There is a continuous transition at a critical temperature T_c defined by

$$\left. \frac{d^2}{dr^2} \mathcal{V}^{\text{SU}(2)}(T_c, r) \right|_{r=\pi} = 0, \quad (33)$$

with $\tilde{m}_c = m/T_c$. A simple calculation yields⁷

$$(d-1) \int_q n'_{\text{FD}}(\tilde{\epsilon}_q) \Big|_{\tilde{m}_c} = \int_q n'_{\text{FD}}(q) \equiv -\frac{\Omega_{d-1}}{(2\pi)^{d-1}} h_d, \quad (34)$$

where $n_{\text{FD}}(x) = (e^x + 1)^{-1}$ and the prime is a derivative. The evaluation of the massless integral h_d is straightforward and gives, for instance, $h_2 = 1/2$, $h_3 = \ln 2$, $h_4 = \pi^2/6$ and, more generally,

$$h_d = \left(1 - \frac{1}{2^{d-3}}\right) \Gamma(d-1) \zeta(d-2) \quad \text{for } d \geq 3. \quad (35)$$

We find⁸ $\tilde{m}_c \simeq 2.97$ in $d = 4$ and $\tilde{m}_c \simeq 1.72$ in $d = 3$.

⁶This is in line with the general arguments developed in [9, 11].

⁷Note that Eq. (26) involves (using an integration by parts) $f(x) = \text{Re } n_{\text{BE}}(x+ir)$, with $n_{\text{BE}}(x) = (e^x - 1)^{-1}$ the Bose-Einstein distribution function. As emphasized before, r appears as an imaginary chemical potential. In the high temperature phase, $r = 0$ and this reduces to the standard Bose-Einstein function: $f(x) = n_{\text{BE}}(x)$. Instead, in the low temperature, confining phase, $r = \pi$, and the transverse gluons and ghosts obey an abnormal Fermi-Dirac statistics: $f(x) = -n_{\text{FD}}(x)$.

⁸Interestingly, the above analysis implies $\tilde{m}_c = 0$ in $d = 2$: There is no phase transition and the system is always in the symmetric (confining) phase. We mention, however, that the massive model considered here is problematic when $d = 2$ [24, 25].

In principle, estimates for the gluon mass can be inferred from the value of an independent observable, such as the glueball mass, or from gauge-fixed lattice data for, say, the Yang-Mills propagators. In full generality, this mass parameter, e.g., when viewed as effectively arising from a proper account of Gribov copies, may depend on both the temperature and the background field. No lattice calculations exist so far in the LDW gauge used in the present work, but we can use estimates from the $T = 0$ lattice data in the Landau gauge [35], in the spirit of Refs. [24, 25]. This makes sense because the minimum of the background field potential scales as T and vanishes in the vacuum: The LDW gauge thus reduces to the Landau gauge.

In a previous work [29], we could study the temperature dependence of the gluon mass by fitting the gluon and ghost correlators at finite temperature in the Landau gauge to existing lattice data. This showed almost no dependence in the range of temperature considered. Although we have no guaranty that this remains true in the present case, it indicates that an estimate at $T = 0$ is probably not completely off.

The fits performed in Refs. [24, 25] used one-loop propagators which is beyond the order of approximation of the present calculation. To be consistent, we have performed fits using the tree-level vacuum propagators. In $d = 4$ we obtain $m = 710$ MeV, which gives $T_c \simeq 238$ MeV. A direct comparison to values of T_c from lattice studies or from other continuum approaches is difficult because of the issue of properly setting the scale. A typical lattice value [47] is $T_c = 295$ MeV and the most recent value from FRG studies [11] is $T_c = 230$ MeV. Although, as mentioned above, our above estimate is only qualitative, our one-loop calculation falls in the right ballpark.

3.2. First order transition for $N = 3$

We now consider the case $N = 3$. The Cartan subalgebra has two directions, $a = 3, 8$ in the usual Gell-Mann representation. As discussed in Appendix A, the Z_3 -center symmetry puts no constraint in the r_3 -direction. Standard (periodic) gauge transformations in that direction leads to a 4π -periodicity in r_3 . In contrast, the Z_3 -symmetry gives a shorter periodicity of $4\pi/\sqrt{3}$ in the r_8 direction. We thus have

$$\begin{aligned} V^{\text{SU}(3)}(T, r_3, r_8) &= V^{\text{SU}(3)}(T, r_3, r_8 + 4\pi/\sqrt{3}) \\ &= V^{\text{SU}(3)}(T, r_3 + 4\pi, r_8). \end{aligned} \quad (36)$$

As described in the Appendix, charge conjugation invariance and the global color symmetry further imply

$$\begin{aligned} V^{\text{SU}(3)}(T, r_3, r_8) &= V^{\text{SU}(3)}(T, -r_3, -r_8) \\ &= V^{\text{SU}(3)}(T, r'_3, r'_8) = V^{\text{SU}(3)}(T, -r_3, r_8), \end{aligned} \quad (37)$$

where (r'_3, r'_8) is obtained from (r_3, r_8) by a rotation of angle $\pm\pi/3$. It is thus sufficient to study an equilateral triangle of side $4\pi/\sqrt{3}$ in the (r_3, r_8) plane, with vertices at $(0, 0)$ and $(2\pi, \pm 2\pi/\sqrt{3})$. Furthermore, the above symmetries imply that the potential is invariant under rotations which leave this equilateral triangle invariant, such that only a third of it is actually relevant. Finally, this implies that the vertices of the triangle and its center, located at $(4\pi/3, 0)$, are always extrema of the background field potential.

The expression of the tree-level Polyakov loop (18) reads

$$\ell = \frac{1}{3} \left[e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}} \cos(r_3/2) \right], \quad (38)$$

from which one sees that the vertices of the triangle correspond to Z_3 -breaking solutions, with $\ell = \exp(2ni\pi/3)$, $n \in \mathbb{Z}$, whereas its center corresponds to the Z_3 -symmetric solution, with $\ell = 0$. We shall see below that the absolute minimum of the one-loop background field potential lies on the $r_8 = 0$ axis, up to rotations of $\pm\pi/3$ as mentioned above, where the tree-level Polyakov loop is real:⁹

$$\ell = \frac{1 + 2 \cos(r_3/2)}{3}. \quad (39)$$

The calculation of the background field potential follows similar lines as in the SU(2) case. The covariant derivative $i\bar{D}^{ab}$ is block-diagonal with two 1×1 blocks corresponding to the Cartan directions and three 2×2 blocks with similar structures as in the SU(2) case, corresponding to the three SU(2) subgroups of SU(3). The result can thus be written in terms of the function (27). Ignoring, again, a constant contribution, we obtain, for the dimensionless potential $\mathcal{V}^{\text{SU}(3)} \equiv V^{\text{SU}(3)}/T^d$

$$\mathcal{V}^{\text{SU}(3)} \doteq 2 [\mathcal{W}_{\tilde{m}}(0) + \mathcal{W}_{\tilde{m}}(r_3) + \mathcal{W}_{\tilde{m}}(r_+) + \mathcal{W}_{\tilde{m}}(r_-)], \quad (40)$$

where $r_{\pm} = (-r_3 \pm \sqrt{3}r_8)/2$. The first term inside the bracket arises from the two color modes in the Cartan subspace whereas the last three terms are the contributions from the six transverse modes. It is easy to check that the previous expression is invariant under the symmetries described above.

⁹This is consistent with the fact that the Polyakov loop is proportional to $\exp(-\beta F)$, where F is the free energy of a static quark.

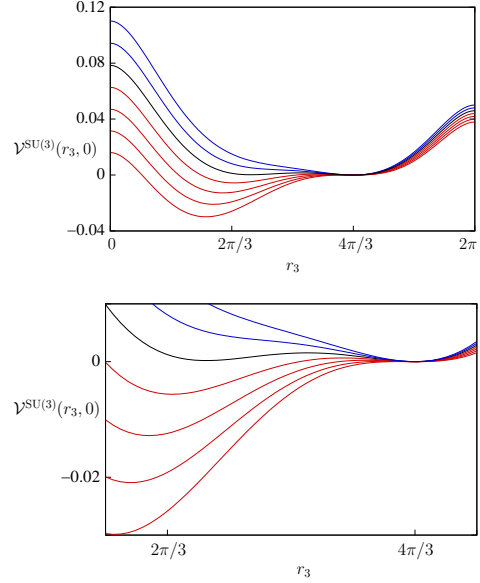


Figure 2: The SU(3) background field potential in the $r_8 = 0$ direction in $d = 4$, for decreasing values of \tilde{m} (increasing temperatures) from top to bottom. The bottom figure is a close-up view around T_c . (color online: $T = T_c$ (black), $T < T_c$ (blue), $T > T_c$ (red))

At high temperatures, one recovers the Weiss potential [46]

$$\mathcal{V}^{\text{SU}(3)} \approx (d-2)[F_0(0) + F_0(r_3) + F_0(r_+) + F_0(r_-)], \quad (41)$$

which has degenerate minima on the vertices of the basic equilateral triangle, corresponding to a deconfined phase. At low temperatures, we get, instead,

$$\mathcal{V}^{\text{SU}(3)} \approx -[F_0(0) + F_0(r_3) + F_0(r_+) + F_0(r_-)] \quad (42)$$

and one easily checks that the absolute minimum lies at the center of the triangle, corresponding to a Z_3 -symmetric, confining phase. A detailed analysis reveals that the transition is first order and that the absolute minimum lies on the $r_8 = 0$ axis (up to the aforementioned discrete rotations) for all temperatures. The potential (40) is plotted as a function of r_3 for $r_8 = 0$ in Fig. 2 for various temperatures. We find the transition temperature $\tilde{m}_c \simeq 2.75$ in $d = 4$. As before, we estimate the gluon mass parameter from fits of SU(3) lattice data for the vacuum propagator in the Landau gauge, again its tree-level expression. In $d = 4$, we find $m = 510$ MeV which gives the estimate $T_c = 185$ MeV. With the same words of caution as in the case $N = 2$, we mention the results from lattice calculations [47], $T_c = 270$ MeV, and from FRG studies [11], $T_c = 275$ MeV.

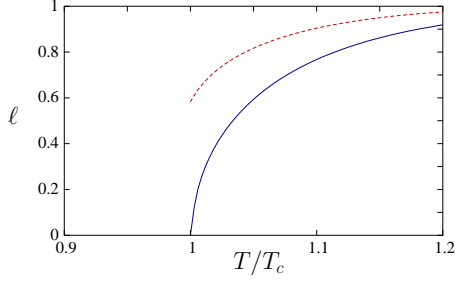


Figure 3: The Polyakov loop as a function of the temperature normalized to the transition temperature for $N = 2$ (solid line) and $N = 3$ (dashed line). (color online)

Our results are summarized in Fig. 3, which shows the temperature dependence of the Polyakov loop for both $N = 2$ and $N = 3$.

4. The LDW gauge on the lattice, Gribov ambiguities and the gluon mass

Let us now briefly discuss the possible relation between the gluon mass introduced in (6) and the Gribov ambiguities of the LDW gauge. Writing the field $A = \bar{A} + a$, the gauge invariance of the Yang-Mills theory can be decomposed as

$$\bar{A}_\mu \rightarrow \bar{A}_\mu^U \quad \text{and} \quad a_\mu \rightarrow U a_\mu U^{-1}, \quad (43)$$

or, equivalently, as

$$\bar{A}_\mu \rightarrow \bar{A}_\mu \quad \text{and} \quad a_\mu \rightarrow a_\mu^U = U a_\mu U^{-1} + \frac{i}{g} U \bar{D}_\mu U^{-1}, \quad (44)$$

where $\bar{D}_\mu U = \partial_\mu U - ig[\bar{A}_\mu, U]$. The LDW condition (2) is covariant under (43) – hence the symmetry (7) of the gauge-fixed effective action – but breaks the symmetry (44).

The transformation $a \rightarrow a^U$ resembles closely the usual gauge transformation with the replacement $\partial_\mu \rightarrow \bar{D}_\mu$. Observe that the LDW gauge condition and the complete gauge-fixed action (3) can be formally obtained from that in the Landau gauge by the same replacement. This can be used as a starting point for a nonperturbative formulation of the LDW gauge, as proposed in [48]. Indeed, it is well-known that the Landau gauge condition can be obtained by extremizing the functional $\int_x \text{tr}\{A_\mu^U A_\mu^U\}$ with respect to U for a given A , which is the basis of efficient numerical implementations in lattice calculations. A simple generalization to the LDW gauge reads [48]

$$F_{\bar{A}}[a, U] = \int_x \text{tr}\{a_\mu^U a_\mu^U\}. \quad (45)$$

It is easy to check that the extrema of this functional with respect to U for fixed a and \bar{A} satisfy the LDW condition (2). The authors of [48] have demonstrated that background field gauges can be efficiently implemented in lattice calculations by means of similar methods as those routinely employed in the (minimal) Landau gauge.

Continuum approaches face the usual Gribov problem, namely the fact that the functional (45) has many extrema (among which many minima). The gauge-fixed action (3) is based on the standard Fadeev-Popov construction which ignores Gribov copies and is, at best, a valid procedure in the high momentum regime. This is not a problem for lattice calculations since it is always possible to pick up one (local) minimum per gauge configuration, as done in the minimal Landau gauge. However, no such procedure can be formulated in the continuum in terms of a local and renormalizable action [49].

Instead, the recent proposal of Refs. [26, 28] consists in taking a particular average over all copies of a given gauge orbit. It was shown in [26] for the case of the Landau gauge that such a gauge-fixing procedure can be formulated in terms of a local, perturbatively renormalizable action in $d = 4$.¹⁰ It was further shown that, for what concerns the calculation of gluon and ghost correlators, this gauge-fixed theory is perturbatively equivalent to the CFFT model mentioned in the introduction. In fact, the averaging procedure in [26] involves the Landau gauge extremization functional mentioned above, which eventually leads to the effective gluon mass term.¹¹

It is straightforward to generalize the proposal of [26] to the LDW gauge by using (45) in place of the Landau gauge extremization functional. Eventually, this amounts to the replacement $\partial_\mu \rightarrow \bar{D}_\mu$ described previously. In particular, it can be shown that the main results hold: the gauge-fixed theory can be formulated in terms of a local action, perturbatively renormalizable in $d = 4$, and it effectively reduces to the massive action (6) for what concerns the calculation of gluon and ghost correlators and, consequently, of gauge invariant observables.

5. Conclusions

We have shown that the phase structure of $SU(N)$ Yang-Mills theories can be described in a perturbative

¹⁰This has been generalized in [28] to a broader class of (nonlinear) covariant gauges.

¹¹More precisely, the effective bare mass has to be sent to zero together with the (continuum) limit of vanishing bare coupling. This can be done by keeping the renormalized mass finite.

set-up at the first non-trivial, one-loop order. This is a remarkable result. Our approach is based on a simple massive modification of the usual Fadeev-Popov action in the LDW gauge, where the mass term can be related to the issue of properly taking into account Gribov ambiguities. Using values of the mass parameter from fits of lattice data for gluon correlators at vanishing temperature, we obtain rather satisfying results for the transition temperatures for $N = 2$ and $N = 3$ in view of the simplicity of the calculation. We mention that our one-loop result compares qualitatively well with those of [9, 10] using the FRG approach. This demonstrates that the main nonperturbative physics associated with the deconfinement transition – and thus with the confinement of static quarks – is well captured by a simple gluon mass term and that a perturbative treatment around the massive theory is adequate.

This has various welcome consequences: First, the relevant physics can be efficiently described by simple (perturbative) calculations. For instance, an interesting extension of the present work is to include quarks at finite baryonic chemical potential at one loop order. This will be presented elsewhere. Second, the approximation can be systematically improved by going to higher orders. For instance, it is a hard but doable calculation to improve on the results presented here by going to two-loop order. Preliminary results show that the critical temperature in the SU(2) case comes closer to its actual value and that the thermodynamic pressure is better behaved as a function of the temperature. This, also, will be the object of a separate publication.

We mention that we have computed the pressure (15) as a function of temperature. The one-loop result has several unwanted features such as the fact that it behaves as T^4 (in $d = 4$) at low temperatures and that it briefly decreases with temperature around the phase transition. Our preliminary two-loop results seem to resolve this second problem, see [50] for the SU(2) case. Contrarily to the Polyakov loop, whose behavior is governed by symmetry, the pressure is sensitive to the full Yang-Mills dynamics, which probably explains why it is more difficult to compute accurately.

Finally, we emphasize that the ability of the present approach to efficiently capture the essential aspects of the phase transition is deeply related to the fact that the center symmetry is manifest at each step of the calculation. In contrast, the Z_N symmetry is not explicit in the Landau gauge. This might be related to the poor convergence of lattice data for the chromoelectric gluon propagator in the Landau gauge near the transition temperature [40]. It would be interesting to study the possibility of allowing for a nontrivial background temporal

gluon field in lattice calculations of Yang-Mills correlators at finite temperature by implementing a discrete version of the LDW gauge, e.g., along the lines of [48].

Acknowledgements

We would like to thank L. Fister, T. Mendes and J.M. Pawłowski for interesting discussions on topics related to this work. We also acknowledge support from the ECOS cooperation program and financial support from the PEDECIBA program.

Appendix A. Symmetries

For completeness we briefly recall the basic constraints from the Z_N symmetry and the charge conjugation invariance. We consider gauge transformations of the form $U(\tau, \mathbf{x}) = \exp \{i\tau\varphi/\beta\}$, with $\varphi = \varphi_a t^a_C$ an element of the Cartan algebra. For appropriate values of φ , these preserve both the β -periodicity in imaginary time and the spatial homogeneity of the background field and leave it in the temporal direction and in the Cartan algebra. Under such a gauge transformation, the parameters $r_a = \beta g \bar{A}_0^a$ get shifted as $r_a \rightarrow r_a + \varphi_a$. Depending on the allowed values of the angles φ_a , the background field gauge invariance thus leads to periodicity properties of the potential (14). Below we examine the constraints from Z_N transformations, which correspond to gauge transformations with twisted boundary conditions, $U^\dagger(\beta)U(0) = e^{2i\pi/N} \mathbb{1}$.

We also exploit the invariance of the (gauge-fixed) theory under the charge-conjugation transformation $(A_\mu^a t^a) \rightarrow -(A_\mu^a t^a)^*$ which leads to a parity symmetry $r_a \rightarrow -r_a$ of the potential (14) in the Cartan algebra.

In the case $N = 2$, the Cartan algebra has a single generator $t_3 = \frac{1}{2}\text{diag}(1, -1)$. One easily checks that periodic gauge transformations, such that $U^\dagger(\beta)U(0) = \mathbb{1}$ correspond to $\varphi_3 = 4\pi$ whereas twisted (Z_2) transformations lead to a shorter 2π -periodicity. Together with charge conjugation invariance, this leads to the symmetry property (19).

In the case $N = 3$, the two generators of the Cartan algebra can be chosen as $t_3 = \frac{1}{2}\text{diag}(1, -1, 0)$ and $t_8 = \frac{1}{2\sqrt{3}}\text{diag}(1, 1, -2)$. The presence of the vanishing eigenvalue of t_3 prevents the possibility of a twisted gauge transformation. Only standard periodic gauge transformations are possible in the r_3 -direction, which leads to a 4π -periodicity. In the r_8 -direction, twisted Z_3 transformations are possible and lead to a shorter $4\pi/\sqrt{3}$ -periodicity. The other symmetries of the potential follow from charge conjugation invariance and the

global color symmetry. They imply in particular that $r_3 = 0$, $r_8 = 0$, $r_8 = \pm r_3/\sqrt{3}$ and $r_8 = \pm r_3\sqrt{3}$ are symmetry axes.

References

- [1] B. Svetitsky, Phys. Rept. **132** (1986) 1.
- [2] J. Fingberg, U. M. Heller and F. Karsch, Nucl. Phys. B **392**, 493 (1993).
- [3] A. M. Polyakov, Phys. Lett. B **72**, 477 (1978).
- [4] O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, Phys. Lett. B **543**, 41 (2002).
- [5] B. Lucini, M. Teper and U. Wenger, JHEP **0502**, 033 (2005).
- [6] J. Greensite, Phys. Rev. D **86**, 114507 (2012).
- [7] D. Smith, A. Dumitru, R. Pisarski and L. von Smekal, Phys. Rev. D **88**, no. 5, 054020 (2013).
- [8] F. Marhauser and J. M. Pawłowski, arXiv:0812.1144 [hep-ph].
- [9] J. Braun, H. Gies and J. M. Pawłowski, Phys. Lett. B **684** (2010) 262.
- [10] J. Braun, A. Eichhorn, H. Gies and J. M. Pawłowski, Eur. Phys. J. C **70** (2010) 689.
- [11] L. Fister and J. M. Pawłowski, Phys. Rev. D **88**, 045010 (2013).
- [12] D. Epple, H. Reinhardt and W. Schleifenbaum, Phys. Rev. D **75**, 045011 (2007).
- [13] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Mod. Phys. Lett. A **23**, 1105 (2008).
- [14] C. S. Fischer, Phys. Rev. Lett. **103**, 052003 (2009).
- [15] C. S. Fischer and J. A. Mueller, Phys. Rev. D **80**, 074029 (2009).
- [16] H. Reinhardt and J. Heffner, Phys. Lett. B **718** (2012) 672; Phys. Rev. D **88** (2013) 4, 045024.
- [17] K. Fukushima and K. Kashiwa, Phys. Lett. B **723** (2013) 360.
- [18] D. Zwanziger, Nucl. Phys. B **323**, 513 (1989).
- [19] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. D **78** (2008) 065047.
- [20] D. Zwanziger, Phys. Rev. Lett. **94**, 182301 (2005).
- [21] D. Zwanziger, Phys. Rev. D **76**, 125014 (2007).
- [22] K. Fukushima and N. Su, Phys. Rev. D **88**, 076008 (2013).
- [23] F. Canfora, P. Pais and P. Salgado-Rebolledo, Eur. Phys. J. C **74**, 2855 (2014).
- [24] M. Tissier and N. Wschebor, Phys. Rev. D **82** (2010) 101701.
- [25] M. Tissier and N. Wschebor, Phys. Rev. D **84** (2011) 045018.
- [26] J. Serreau and M. Tissier, Phys. Lett. B **712** (2012) 97.
- [27] M. Peláez, M. Tissier and N. Wschebor, Phys. Rev. D **88** (2013) 125003.
- [28] J. Serreau, M. Tissier and A. Tresmontant, Phys. Rev. D **89** (2014) 125019.
- [29] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys. Rev. D **89**, 105016 (2014).
- [30] M. Peláez, M. Tissier and N. Wschebor, Phys. Rev. D **90** (2014) 065031.
- [31] G. Curci and R. Ferrari, Nuovo Cim. A **32**, 151 (1976).
- [32] E. S. Fradkin and I. V. Tyutin, Phys. Lett. B **30** (1969) 562.
- [33] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6** (1973) 1181.
- [34] J. V. Jose, L. P. Kadanoff, S. Kirkpatrick and D. R. Nelson, Phys. Rev. B **16**, 1217 (1977).
- [35] A. Cucchieri and T. Mendes, PoS FACESQCD (2010) 007.
- [36] R. Aouane, V. G. Bornyakov, E. M. Ilgenfritz, V. K. Mitjushkin, M. Müller-Preussker and A. Sternbeck, Phys. Rev. D **85** (2012) 034501.
- [37] A. Maas, J. M. Pawłowski, L. von Smekal and D. Spielmann, Phys. Rev. D **85** (2012) 034037.
- [38] P. J. Silva, O. Oliveira, P. Bicudo and N. Cardoso, Phys. Rev. D **89**, 074503 (2014).
- [39] A. Cucchieri and T. Mendes, PoS LATTICE **2011** (2011) 206.
- [40] T. Mendes and A. Cucchieri, PoS LATTICE **2013**, 456 (2014).
- [41] L. Fister and J. M. Pawłowski, arXiv:1112.5440 [hep-ph].
- [42] M. Tissier and N. Wschebor, Phys. Rev. D **79** (2009) 065008.
- [43] D. Binosi and A. Quadri, Phys. Rev. D **88** (2013) 085036.
- [44] C. Gattringer, Phys. Rev. Lett. **97**, 032003 (2006).
- [45] S. Weinberg, Cambridge, UK: Univ. Pr. (1996) 489 p.
- [46] N. Weiss, Phys. Rev. D **24** (1981) 475.
- [47] B. Lucini and M. Panero, Phys. Rept. **526** (2013) 93.
- [48] A. Cucchieri and T. Mendes, Phys. Rev. D **86** (2012) 071503.
- [49] V. N. Gribov, Nucl. Phys. B **139** (1978) 1.
- [50] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys. Rev. D **91**, no. 4, 045035 (2015).